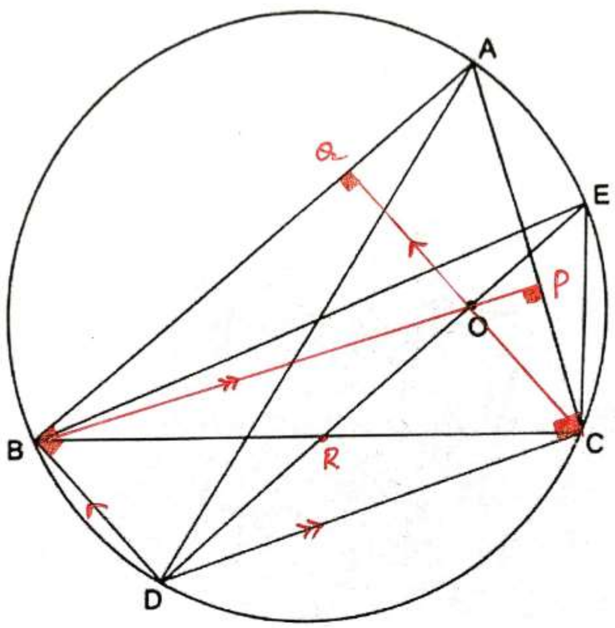
**Given :** ' O' is orthocentre

**To Prove :**

$$BE^2 + EC^2 + CD^2 + BD^2 = 2DE^2$$




A 'O' is orthocentre CQ ⊥ AB & BP⊥ $AC$

And As AD is diameter DB ⊥ AB & DC ⊥ $AC$ [ Angle in Semi circle is 90°

∴ CQ ∥ DB & BP ∥ DC

⇒ BOCD is a parallelogram in which OD & BC are diagonals.

<u>WKT</u>

Diagnols of parallelogram bisects each other

⇒ R is the midpoint BC & DO.

⇒ $BR = RC$ & DR = RO ----------------- (1)

Now In Δ EBC; ER is median

∴ $BE^2 + EC^2 = 2ER^2 + \frac{1}{2}BC^2$ [ Apollonius theorem] ----- (2)

In Δ DBC; DR is median

$\therefore BD^2 + DC^2 = 2\,DR^2 + \frac{1}{2}BC^2$ [ Apollonius theorem] ------(3)

Adding (2) & (3)

$BE^2 + EC^2 + CD^2 + BD^2 = 2ER^2 + 2\,DR^2 + BC^2$

$= [2ER^2 + DR^2] + BC^2$

$= 2[(ER + DR)^2 - 2ER.DR] + (2BR)^2$

$= 2[DE^2 - 2ER.DR] + 4BR^2$

$BE^2 + EC^2 + CD^2 + BD^2 = 2DE^2 - 4ER \times DR + 4BR \times BR$ -------(4)

$BR \times RC = ER \times DR$ [Chords intersecting inside the circle]

As R is the midpoint of BC [ ie. BR = RC]

⇒ BR x BR = ER x DR ------------------------ (5)

Substitute above in equation (4)

$BE^2 + EC^2 + CD^2 + BD^2 = 2DE^2 - 4ER \times DR + 4\,BR \times BR$

$= 2DE^2 - 4\cancel{ER} \times DR + 4\,\cancel{ER} \times DR$

$= 2DE^2$

-------- Hence Proved

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- HARA GOPAL